



D&D Resources Ltd
Mathematics Exam Preparation Made Easy

Differentiation - 91578

Practice External Assessments 3

DIFFERENTIATION

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NCEA 3 Maths

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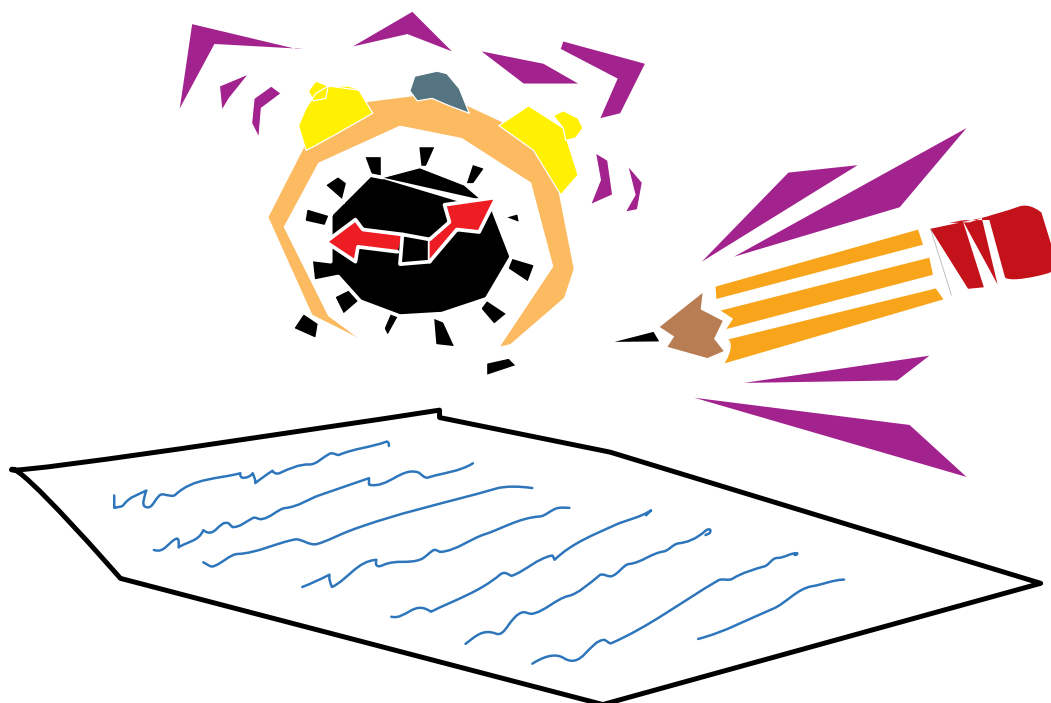
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Prior to attempting the practice assessments we suggest you download our Differentiation, 'Readiness Check', from the D & D Resources website. It comprises a series of questions together with answers which you can use as a 'warm up' before undertaking the assessments in this booklet.

The questions in the practice assessments are NOT in order of difficulty. Attempt all questions or you may not provide enough evidence to achieve the required standard.

Achievement Standard

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Apply differentiation methods in solving problems

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

- ◆ This achievement standard is derived from Level 8 of The New Zealand Curriculum and is related to the achievement objectives
 - ❖ Identify discontinuities and limits of functions.
 - ❖ Choose and apply a variety of differentiation techniques to functions and relations using analytical methods.
- ◆ Apply differentiation methods in solving problems involves:
 - ❖ selecting and using methods
 - ❖ demonstrating knowledge of concepts and terms
 - ❖ communicating using representations.
- ◆ Relational thinking involves one or more of:
 - ❖ selecting and carrying out a logical sequence of steps
 - ❖ connecting different concepts or representations
 - ❖ demonstrating understanding of concepts
 - ❖ forming and using a model;
 and also relating findings to a context, or communicating thinking using appropriate mathematical statements.
- ◆ Extended abstract thinking involves one or more of:
 - ❖ devising a strategy to investigate or solve a problem
 - ❖ identifying relevant concepts in context
 - ❖ developing a chain of logical reasoning, or proof
 - ❖ forming a generalisation;
 and also using correct mathematical statements, or communicating mathematical insight.
- ◆ Problems are situations that provide opportunities to apply knowledge or understanding of mathematical concepts and methods. Situations will be set in real-life or mathematical contexts.
- ◆ Methods include a selection from those related to:
 - ❖ derivatives of power, exponential, and logarithmic (base e only) functions
 - ❖ derivatives of trigonometric (including reciprocal) functions
 - ❖ optimisation
 - ❖ equations of normals
 - ❖ maxima and minima and points of inflection
 - ❖ related rates of change
 - ❖ derivatives of parametric functions
 - ❖ chain, product, and quotient rules
 - ❖ properties of graphs (limits, differentiability, continuity, concavity).

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PEA 2

You are advised to spend 60 minutes answering this assessment.

You should show ALL working and answer ALL parts of ALL questions.

QUESTION ONE

- (a) Differentiate $y = \ln(\operatorname{cosec} x)$.

- (b) The amount of money in a school account during the first six months of the school year can be modelled by the equation $M = e^{x/2} - 2x + 3$, $0 \leq x \leq 6$, where M is the amount in hundreds of dollars and x is the month number. During the six month period, what is the minimum amount of money in the school account?

- (c) From the graph of $y = f(x)$ state:

1. For which values of a does $\lim_{x \rightarrow a} f(x)$ not exist?

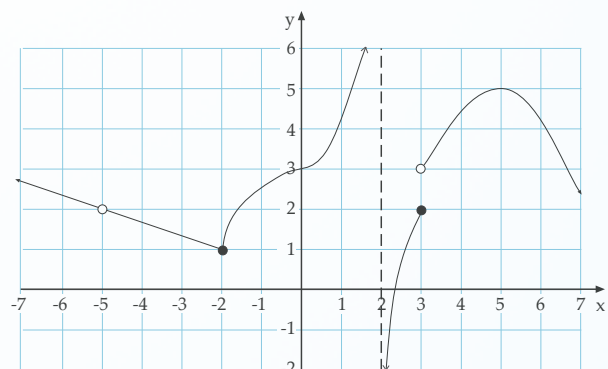
2. List all the x values that meet the following conditions

- i) $f(x)$ is continuous but not differentiable.

ii) $f'(x) = 0$

iii) $f''(x) > 0$

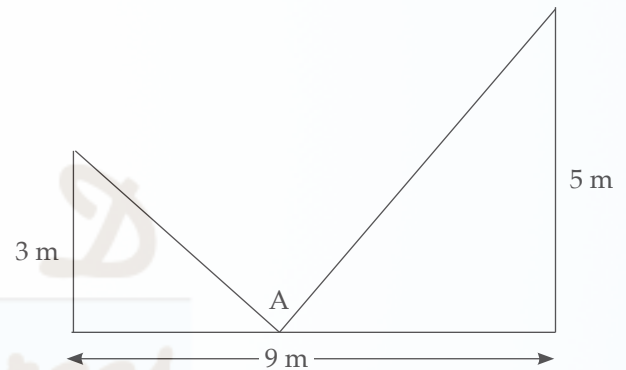
3. What is the value of $f(-5)$?



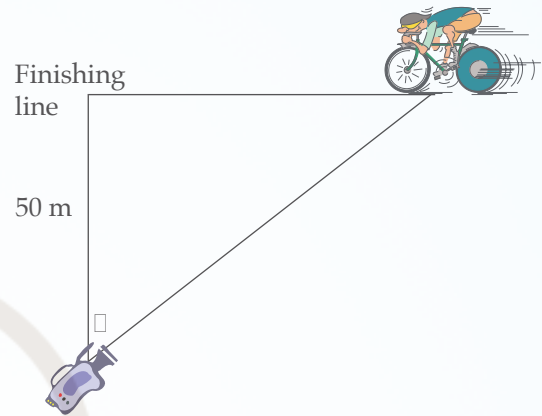
- (d) The height of a toy rocket t seconds after launch can be modelled by the equation $h = 0.5t^2 \ln t + 5$, where h is the height in metres and t the time in seconds after launch. How many seconds after launch does the toy rocket reach its minimum height?



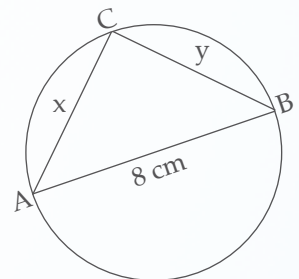
- (e) Two vertical masts, one 3 m high and one 5 m high, stand 9 m apart as shown in the diagram. Each mast is to be stayed by means of a wire to an anchor point A. How far from the shorter mast should the anchor be placed so that the least amount of wire is required?



- (d) A spectator is video-taping a cycling race from a stand 50 metres away from the track. The lead cyclist is travelling at a steady speed of 18 ms^{-1} . Calculate how fast the camera angle is changing in radians per second when the leading cyclist is 20 metres away from the finishing line.



- (e) A circle has a diameter AB of length 8 cm. C is a point on the circumference. AC has length x and BC has length y . Show that triangle ABC will be isosceles if the perimeter is a maximum and find the maximum perimeter.



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PEA 5

Quest.	Evidence	Achievement	Merit	Excellence
		Apply differentiation methods in solving problems	Apply differentiation methods, using relational thinking, in solving problems	Apply differentiation methods, using extended abstract thinking, in solving problems
ONE				
(a)	$\frac{dy}{dx} = -4x \cos(x^2 - 1)$	• correct derivative.		
(b)	$f = 4x^2$ $f' = 8x$ $g = x - 3$ $g' = 1$ $f'(x) = \frac{8x(x-3) - 4x^2}{(x-3)^2} = \frac{4x^2 - 24x}{(x-3)^2}$ Stationary when $4x^2 - 24x = 0$ $x = 0, 6$.	• correct solution with correct derivative.		
(c)	$\frac{dd}{dt} = 10 \text{ cms}^{-1}$ $d^2 = x^2 + x^2 + x^2 = 3x^2$ $d = \sqrt{3}x$ $\frac{dd}{dx} = \sqrt{3}$ $\frac{dx}{dt} = \frac{dx}{dd} \times \frac{dd}{dt} = \frac{10}{\sqrt{3}}$ or 5.77 cms^{-1}	• correct expression for $\frac{dd}{dx}$.	• correct solution with correct derivatives.	
(d)	$\frac{dx}{dt} = 3t^2 - 1$ $\frac{dy}{dt} = 2t + 1$ $\frac{dy}{dx} = \frac{2t+1}{3t^2-1}$ Tangent is parallel to x axis when $\frac{2t+1}{3t^2-1} = 0$ $2t + 1 = 0, t = -\frac{1}{2}$ $x = \frac{3}{8}, y = \frac{-1}{4}$	• correct expression for $\frac{dy}{dx}$.	• correct solution with correct derivatives.	
(e)	Length = $6 + 8 \cos \theta$ and width = $4 \sin \theta$ $\text{Area} = \frac{12 + 8 \cos \theta}{2} \times 4 \sin \theta$ $= 24 \sin \theta + 16 \cos \theta \sin \theta$ $= 24 \sin \theta + 8 \sin 2\theta$ $\frac{dA}{d\theta} = 24 \cos \theta + 16 \cos 2\theta$ $\frac{dA}{d\theta} = 24 \cos \theta + 16(2 \cos^2 \theta - 1)$ $\frac{dA}{d\theta} = 32 \cos^2 \theta + 24 \cos \theta - 16$ $8(4 \cos^2 \theta + 3 \cos \theta - 2) = 0$ $\cos \theta = 0.4254$ $\theta = 1.1314$ Width of pen = 3.62 m. Length of pen = 9.4 m.		• correct derivative for $\frac{dA}{d\theta}$.	• correct solution with correct derivative.